

1. (a) [1 分]

$$B_\nu(T) = \frac{2h\nu}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

$$\nu\lambda = c \Rightarrow d\nu = -\frac{c}{\lambda^2} d\lambda$$

$$\begin{aligned} B_\nu d\nu = B_\lambda d\lambda \Rightarrow B_\lambda &= \frac{C}{\lambda^2} B_\nu = \frac{C}{\lambda^2} \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \\ &= \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \end{aligned}$$

(b) [1 分]

$$\text{设 } x = \frac{hc}{\lambda kT}, \lambda = \frac{hc}{kT} \frac{1}{x}$$

$$\Rightarrow B_\lambda = \frac{2}{h^4 c^3} k^5 T^5 \frac{x^5}{e^x - 1}$$

$$\Rightarrow \frac{\partial B_\lambda}{\partial x} = 0$$

$$\Rightarrow \frac{\partial}{\partial x} \left(\frac{x^5}{e^x - 1} \right) = 0 \Rightarrow \frac{x e^x}{e^x - 1} = 5$$

$$\Rightarrow x \approx 4.965 = \frac{hc}{\lambda kT}$$

$$\Rightarrow \lambda T = 2898 \mu\text{mK}$$

(c) [1 分]

$$h\nu \ll kT$$

$$e^{\frac{h\nu}{kT}} - 1 = 1 + \frac{h\nu}{kT} - 1 = \frac{h\nu}{kT}$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{kT}{h\nu} = \frac{2kT\nu^2}{c^2}$$

2. [2 分]

四个影响：

(a) 消光：大气中的气体、尘埃颗粒等对天体的辐射产生吸收和散射，减弱天体辐射的强度以及改变天体的能谱。

(b) 折射：大气对天体辐射产生折射，改变传播方向，电离层对射电长波的全反射使得地面无法观测长波射电。

(c) 辐射：地球大气产生热辐射以及荧光发射线。

(d) 湍动：大气因速度、温度、压强等变化对星光造成扰动，影响成像质量。

3. [2 分]

两发动机成功：

$$\begin{aligned} P_2 &= P(m=1, p=0.5, n=2) + P(m=2, p=0.5, n=2) \\ &= 2 \times 0.5 \times 0.5 + 0.5^2 \\ &= 75\% \end{aligned}$$

四发动机成功：

$$\begin{aligned} P_4 &= P(m=2, p=0.5, n=2) + P(m=3, p=0.5, n=2) + P(m=4, p=0.5, n=2) \\ &= 6 \times 0.5^2 \times 0.5^2 + 4 \times 0.5^3 \times 0.5 + 0.5^4 \\ &= 69\% \end{aligned}$$

$P_4 < P_2$ ，四发动机不比两发动机安全。

4. (a) [0.5 分] $S_{\text{sky}} = 1500\pi \times (1.5)^2 \times 100 = 1060290$ 个

$$\text{噪声 } \sigma_{\text{sky}} = S_{\text{sky}}^{0.5} = 1030 \text{ 个}$$

$$\text{SNR} = S_{\text{sky}} / \sigma_{\text{sky}} = 1030$$

(b) [0.5 分] $S_{\text{tot}} = S_{\text{sky}} + S_{\text{star}}$,

$$\text{其中 } S_{\text{sky}} = 1060290 \text{ 个}, S_{\text{star}} = 100 \times 100 = 10000 \text{ 个}$$

$$S_{\text{tot}} = 1070290 \text{ 个}$$

$$\text{噪声 } \sigma_{\text{tot}} = S_{\text{tot}}^{0.5} = 1034 \text{ 个}$$

$$\text{SNR} = S_{\text{tot}} / \sigma_{\text{tot}} = 1034$$

(c) [1 分] $S_{\text{star}} = S_{\text{tot}} - S_{\text{sky}} = 10000$ 个

$$\sigma_{\text{star}} = \sqrt{\sigma_{\text{tot}}^2 + \sigma_{\text{sky}}^2} = 1460$$

$$\text{SNR} = S_{\text{star}} / \sigma_{\text{star}} = 6.85$$

(4) [1 分] $S_{\text{sky}} = 1500\pi \times (1.5)^2 t, S_{\text{star}} = 100t$

$$S_{\text{tot}} = S_{\text{sky}} + S_{\text{star}}$$

$$\sigma_{\text{star}} = \sqrt{\sigma_{\text{tot}}^2 + \sigma_{\text{sky}}^2} = \sqrt{S_{\text{tot}} + S_{\text{sky}}}$$

$$\text{SNR} = \frac{S_{\text{star}}}{\sigma_{\text{star}}} = \frac{100t}{\sqrt{S_{\text{star}} + 2S_{\text{sky}}}} = \frac{100}{\sqrt{100 + 3000\pi \times 1.5^2}} \sqrt{t} = 100$$

$$0.68\sqrt{t} = 100$$

$$t = 21626.3 \text{ sec}$$